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# COMMON FIXED- POINT THEOREM IN NON-ARCHIMEDEAN INTUITIONISTIC FUZZY METRIC SPACE BY USING SUBCOMPATIBLE MAPS OF TYPE (α) WITH SIX MAPS

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**Abstract:**

In this paper, we have generalized the result of Ferhan Sola Erduran [15] and Anupama Gupta [3] using sub compatible map of type (α) and subsequent continuity. We established a common fixed-point theorem for six maps.

**Keywords:** Weak non- Archimedean intuitionistic fuzzy metric space, subcompatible maps of type (α) & (β), common fixed point.

## Introduction

The fuzzy set was present and explains by Zadeh [19] & fuzzy metric space introduced by Michalak and Kramosil [11], the notion of fuzzy metric space was modified by George and Veeramani [8] in distinct ways. Vasuki verified fixed point theorem for R-weakly commuting maps. Pant found the new conception of common fixed-point theorems.

Intuitionistic fuzzy metric space defined by Alaca et.al.[1] with continuous t-norm and continuous t-conorm. Compatible maps, compatible maps of type (α) & (β) introduced by Turkoglu et.al.[18] in intuitionistic fuzzy metric space.

Lately, Erduran [17] established the concept of weak non- Archimedean intuitionistic fuzzy metric space.

## Main Result

**Theorem:** Let A, B, S, T, P and Q be self-maps of a weak non-Archimedean intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ and let the pair (P, AB) and (Q, ST) are subcompatible maps of type

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(α) and subsequentially continuous. If

$$M(Px,Qy,t) * [M(ABx,Px,t) \,.\, M(STy,Qy,t)] \geq$$
$$\varphi\, [\min\{M(ABx,Px,t), M(ABx,STy,t)\}, \left\{\tfrac{3}{2}(M(ABx,Qy,t) + M(STy,Px,t)\}\right] \qquad (1)$$
$$N(Px,Qy,t) * [N(ABx,Px,t) \,.\, N(STy,Qy,t)] \leq$$
$$\emptyset\, [\max\{N(ABx,Px,t), N(ABx,STy,t)\}, \left\{\tfrac{3}{2}(N(ABx,Qy,t) + N(STy,Px,t)\}\right] \qquad (2)$$

For all x, y ∈ X, t > 0, where $\varphi, \emptyset : [0, 1] \to [0, 1]$ are continuous functions such that $\varphi(s) > s$ and $\emptyset(s) < s$ for each s ∈ (0, 1). Then A, B, S, T, P and Q have a unique fixed point in X.

**Proof.** Since the pairs (P, AB) and (Q, ST) are subcompatible maps of type (α) and subsequentially continuous, then there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} ABx_n = z, z \in X$ and satisfy

$$\lim_{n\to\infty} M(PABx_n, ABABx_n, t) = M(Pz, ABz, t) = 1, \lim_{n\to\infty} N(PABx_n, ABABx_n, t) = N(Pz, ABz, t) = 0,$$
$$\lim_{n\to\infty} M(ABPx_n, PPx_n, t) = M(ABz, Pz, t) = 1, \qquad \lim_{n\to\infty} N(ABPx_n, PPx_n, t) = N(ABz, Pz, t) = 0,$$

$\lim_{n\to\infty} Qy_n = \lim_{n\to\infty} STy_n = w, w \in X$ and satisfy

$$\lim_{n\to\infty} M(QSTy_n, STSTy_n, t) = M(Qw, STw, t) = 1, \lim_{n\to\infty} N(QSTy_n, STSTy_n, t) = N(Qw, STw, t) = 0,$$
$$\lim_{n\to\infty} M(STQy_n, QQy_n, t) = M(STw, Qw, t) = 1, \qquad \lim_{n\to\infty} N(STQy_n, QQy_n, t) = N(STw, Qw, t) = 0,$$

Therefore, Pz = ABz and Qw = STw, that is z is a coincidence point of P and AB; w is a coincidence point of Q and ST.

Now, we prove that $x = z$. By using (1) and (2) for $x = x_n$ and $y = y_n$, we get

$$M(Px_n, Qy_n, t) * [M(ABx_n, Px_n, t) \,.\, M(STy_n, Qy_n, t)]$$
$$\geq \varphi\, [\min\{M(ABx_n, Px_n, t), M(ABx_n, STy_n, t)\}, \{\frac{3}{2}(M(ABx_n, Qy_n, t) + M(STy_n, Px_n, t)\}]$$
$$N(Px_n, Qy_n, t) * [N(ABx_n, Px_n, t) \,.\, N(STy_n, Qy_n, t)]$$
$$\leq \emptyset\, [\max\{N(ABx_n, Px_n, t), N(ABx_n, STy_n, t)\}, \{\frac{3}{2}(N(ABx_n, Qy_n, t) + N(STy_n, Px_n, t)\}]$$

Taking the limit as n → ∞, we have

$$M(z,w,t) * [M(z,z,t) \,.\, M(w,w,t)]$$
$$\geq \varphi\, [\min\{M(z,z,t), M(z,w,t)\}, \{\frac{3}{2}(M(z,w,t) + M(w,z,t)\}]$$
$$N(z,w,kt) * [N(z,z,t) \,.\, N(w,w,t)]$$
$$\leq \emptyset\, [\max\{N(z,z,t), N(z,w,t)\}, \{\frac{3}{2}(N(z,w,t) + N(w,z,t)\}]$$

That is

$$M(z,w,kt) \geq \varphi\, M(w,z,t)$$
$$N(z,w,kt) \leq \emptyset\, N(w,z,t)$$

which yield z = w.

Again using (1) and (2) for $x = z$ and $y = y_n$, we obtain

$$M(Pz, Qy_n, t) * [M(ABz, Pz, t) \,.\, M(STy_n, Qy_n, t)] \geq \varphi\, [\min\{M(ABz, Pz, t), M(ABz, STy_n, t)\}, \{\frac{3}{2}(M(ABz, Qy_n, t) + M(STy_n, Pz, t)\}]$$

$$N(Pz, Qy_n, t) * [N(ABz, Pz, t) \,.\, N(STy_n, Qy_n, t)] \leq \emptyset\, [\max\{N(ABz, Pz, t), N(ABz, STy_n, t)\}, \{\frac{3}{2}(N(ABz, Qy_n, t) + N(STy_n, Pz, t)\}]$$

Taking the limit as n → ∞, we have

$$M(Pz, w, t) * [M(ABz, Pz, t) \,.\, M(w, w, t)] \geq \varphi\, [\min\{M(ABz, Pz, t), M(ABz, w, t)\}, \{\frac{3}{2}(M(ABz, w, t) + M(w, Pz, t)\}]$$

$$N(Pz, w, t) * [N(ABz, Pz, t) \,.\, N(w, w, t)] \leq \emptyset\, [\max\{N(ABz, Pz, t), N(ABz, w, t)\}, \{\frac{3}{2}(N(ABz, w, t) + N(w, Pz, t)\}]$$

That is

$$M(Pz, w, t) \geq \varphi\, M(Pz, w, t)$$
$$N(Pz, w, t) \leq \emptyset\, N(Pz, w, t)$$

Which yield $Pz = w = z$

Similarly, if we Again using (1) and (2) for $x = x_n$ and $y = w$ , we obtain

$$M(z, Qw, t) \geq \varphi\, M(z, Qw, t)$$
$$N(z, Qw, t) \leq \emptyset\, N(z, Qw, t)$$

Which yield $Qw = z = w$.

Therefore $z = w$ is common fixed point of A, B, S, T, P and Q.

**Uniqueness: -** Suppose that there exists another fixed point $u$ of A, B, S, T, P and Q. then from (1) and (2), we have

$$M(Pz, Qu, t) * [M(ABz, Pz, t) \,.\, M(STu, Qu, t)] \geq \varphi\, [\min\{M(ABz, Pz, t), M(ABz, STu, t)\}, \{\frac{3}{2}(M(ABz, Qu, t) + M(STu, Pz, t)\}]$$

$$M(Pz, Qu, t) \geq \varphi\, [\min\{1, M(Pz, Qu, t)\}, \{\tfrac{3}{2}(M(Pz, Qu, t) + M(Qu, Pz, t)\}]$$

$$M(Pz, Qu, t) \geq \varphi\, M(Pz, Qu, t)$$

And

$$N(Pz, Qu, t) * [N(ABz, Pz, t) \,.\, N(STu, Qu, t)] \leq \emptyset\, [\max\{N(ABz, Pz, t), N(ABz, STu, t)\}, \{\frac{3}{2}(N(ABz, Qu, t) + N(STu, Pz, t)\}]$$

$$N(Pz, Qu, t) \leq \emptyset\, [\max\{1, N(Pz, Qu, t)\}, \{\tfrac{3}{2}(N(Pz, Qu, t) + N(Qu, Pz, t)\}]$$

$$N(Pz, Qu, t) \leq \emptyset\, N(Pz, Qu, t)$$

Which yield $z = u$ therefore uniqueness follows.

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